

Computer Graphics

Lecture 15

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Last Class

- **Combining Transformations**
- **Affine versus Rigid body Transformations**
- **Homogenous Transformations**

Today's Agenda

- **Homogeneous transformations**
- **Types of Transformations**
 - **Linear Transformations**
 - **Affine Transformations**
 - **Projective Transformations**

Homogenous Coordinate System

- **Basic 2D transformations in 3D are**
 - **Translation**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scaling as 3x3

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation as 3x3

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear as 3x3

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & h_x & 0 \\ h_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Linear Transformations

- **Combination of Scaling, rotation and shear are linear transformations**

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- **Linear Transformations satisfy following**
 - **Origin maps origin**
 - **Lines maps lines**
 - **Parallelism is preserved**
 - **Ratios remain the same**

Affine Transformations

- **Affine transformations are combinations of linear transformations and Translation**

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- **Affine transformations obey the following**
 - **Origin does **not** necessarily map the origin**
 - **Lines maps lines**
 - **Parallelism is preserved**
 - **Ratios remain the same**

Projective Transformations

- **Affine Transformations and projective warps form projective transformation**

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- **Projective Transformations obey the following**
 - **Origin does **not** necessarily map the origin**
 - **Lines maps lines**
 - **Parallelism is **not** preserved**
 - **Ratios remain are **not** the same**
 - **Closed under composition**

Matrix Composition

- **Homogenous transformations can also be combined by Matrix multiplication**

Multiplication Order

- **Scaling, Rotation and then translation**

Summary

- **Homogeneous transformations**
- **Types of Transformations**
 - **Linear Transformations**
 - **Affine Transformations**
 - **Projective Transformations**

References

- **Fundamentals of Computer Graphics Third Edition by Peter Shirley and Steve Marschner**
- **Interactive Computer Graphics, A Top-down Approach with OpenGL (Sixth Edition) by Edward Angel.**